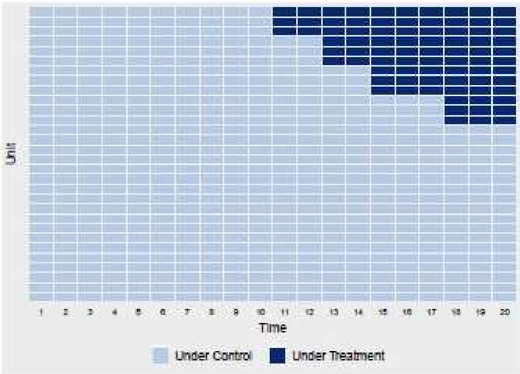
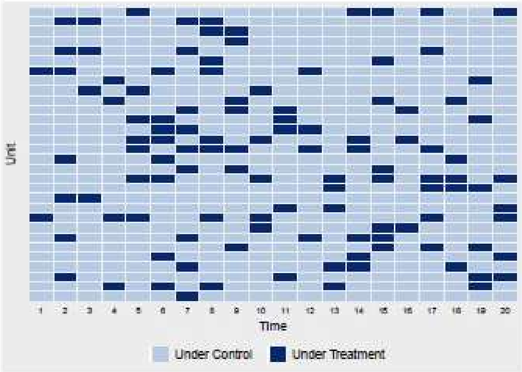
Time Series Cross Sectional Data Analysis

Ⅰ. TSCS 형태

(a) Multi-period DID



(b) Staggered Adoption



(c) General Pattern

Ⅱ. TSCS 접근방법 1 (강한 외생성: Strict Exogeneity Regime)

<examples>: DID, TWFE, LFM, SCM

1. DM-LFM model 일반적 수식(Pang et al., 2022) = dynamic multilevel latent factor model

$$\begin{aligned} y_{it} &= \delta_{it}\omega_{it} + X_{it}^{'}\beta + Z_{it}^{'}\alpha_i + A_{it}^{'}\xi_t + F\gamma^{'} + \epsilon \\ &= \delta_{it}\omega_{it} + X_{it}^{'}\beta_{it} + \gamma_i^{'}f_t + \epsilon_{it},\ if\ \beta_{it} = \beta + \alpha_i + \xi_t, \xi_t = \Phi_\xi \xi_{t-1} + e_t, f_t = \Phi_f f_{t-1} + \nu_t \\ &= \delta_{it}\omega_{it} + X_{it}^{'}(\beta + \alpha_i + \xi_t) + \gamma_i^{'}f_t + \epsilon_{it} \\ &= \delta_{it}\omega_{it} + X_{it}^{'}\beta + X_{it}^{'}(\omega_\alpha \bullet \tilde{\alpha}_i) + X_{it}^{'}(\omega_\xi \bullet \tilde{\xi}_t) + (\omega_\gamma \bullet \tilde{\gamma})^{'}f_t + \epsilon_{it} \end{aligned}$$

$$U = (U_1, U_2, \ldots, U_N) = \Gamma^{'}F$$
$$\Gamma = (\gamma_1, \gamma_2, \ldots, \gamma_N)\,(r \times N)\,(r \ll minN, T); \Gamma_0 = Diag()$$
$$F = (f_1, f_2, \ldots, f_T)^{'}\,(r \times T)$$
$$X_{it} : (T \times p_1)$$
$$Z_{jt}\,(j = 1,2,\ldots,p_2) : (T \times p_2)$$
$$A_{jt}\,(j = 1,2,\ldots,p_3) : (T \times p_3)$$
$$f_j(j = 1,2,\ldots,r) : (T \times r); \Gamma_0 = Diag(\omega_{\gamma_1}^{'}, \omega_{\gamma_2}^{'}, \ldots, \omega_{\gamma_r}^{'})$$
$$\beta : (p_1 \times 1)$$
$$\alpha_i : (p_2 \times 1); H_0 = Diag(\omega_{\alpha_1}^{'}, \omega_{\alpha_2}^{'}, \ldots, \omega_{\alpha_{p2}}^{'})$$
$$\xi = (\xi_1^{'}, \xi_2^{'}, \xi_3^{'}, \xi_4^{'}, \ldots, \xi_T^{'})^{'} : (p_3 \times 1); \Sigma_e = Diag(\omega_{\xi_1}^{'}, \omega_{\xi_2}^{'}, \ldots, \omega_{\xi_{p3}}^{'})$$
$$\nu, e, f_t \sim N(0,1)$$
$$i = (1,2,3,\ldots,N)$$
$$t = (1,2,3,\ldots,T)$$
$$\beta_k|\tau_{\beta_k}^2 \sim N(0, \tau_{\beta_k}^2), \tau_{\beta_k}^2|\lambda_\beta \sim Exp(\frac{\lambda_\beta^2}{2}), \lambda_\beta^2 \sim G(a_1, a_2), k = 1,2,\ldots,p_1$$

2. DiD model(Liu et al., 2020) = fixed effects counterfactual model

$$Z_i = A_i = (1,1,1,\cdots,1)^{'}\ \ \gamma = 0;$$
$$\omega_\alpha = \omega_\beta = \omega_\gamma = 0$$
$$\therefore y_{it} = \delta_{it}\omega_{it} + X_{it}^{'}\beta + \alpha_i + \xi_t + \epsilon_{it}$$

3. SCM model(Abadie et al., 2010) = a factor model = Synthetic Control Method

$$Z_{it} = \varnothing, X_i = A_i time - invariant;$$
$$\therefore y_{it} = \delta_{it}\omega_{it} + X_i^{'}\beta_t + \xi_t + \gamma_i^{'}f_t + \epsilon_{it}$$

==> 비교사례연구에 적합(개체가 적은 특징)

4. Gsyth model(Xu, 2017): Factor-Augmented Approach

$$Z_{it} = A_{it} = \varnothing$$
$$\therefore y_{it} = \delta_{it}\omega_{it} + X_{it}^{'}\beta + \gamma_t^{'}f_t + \epsilon_{it}$$

5. TWTE(Angrist & Pischke, 2009)

$$y_{it} = \delta\omega_{it} + X_{it}^{'}\beta + \alpha_i + \xi_t + \epsilon_{it}$$

=> δ가 고정되어 있다는 점에서, DID의 δ_{it}와 근본적 차이가 있음

α(alpha), β(beta), γ(gamma), δ(delta), ε(epsilon), ξ(xi, 크시)

6. 참고사항

가. DID

(1) Multi-period DID

- Athey and Imbens(2018)
- Egami & Yamauchi(2021)

(2) Staggered Adoption DID

- Goodman-Bacon (2021)
- Callaway & Sant’Anna (2020)

나. DID Extensions

(1) Semiparametric DID Approach (propensity score model, inverse propensity weight)

- Abadie (2005)
- Strezhnev (2018) : Semiparametric + Staggered Adoption
- Sant’Anna & Zhao (2020): doubly robust DID based on the conditional PTA.

다. Twoway Fixed Effects (TWFE)

- 일반적인 패널데이터(figure C)에 매우 적합(Angrist & Pischke, 2009: 236-243)
- 한계 1: δ(처치효과)가 상수라고 가정하면, 강한 외생성(strict exogeneity) 가정을 전제함
- 한계 2: 시간에 따른 역동적인 처치효과를 감안하지 못함. 처치효과가 개체별로 이분산적으로 진화한다면, 처치효과는 편향적이다.
- 한계 3: 지연된 DV의 부재
- 한계 4: no carryover effects.

라. SCM

마. DM-LFM(Factor-Augmented Approach)

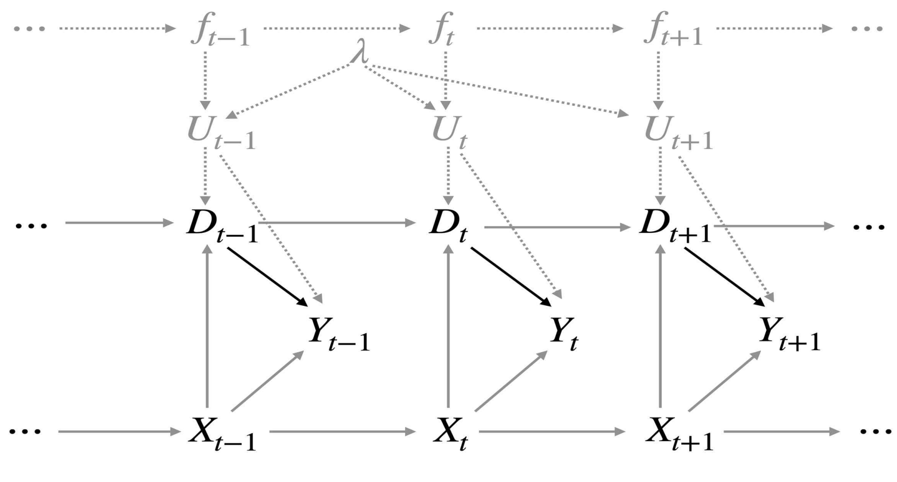


그림 2 DAG for DGPs under Factor-Augmented Approach

DM-LFM 정리(Strict Exogeneity Regime)

1.기본가정. Athey & Imbens(2018)

$i = (1,2,\ ...,N)$
$t = (1,2,\ ...,T)$
$N = N_{co} + N_{tr}$
$a_i \in \{1,2,\ ...,T,c\}$
$N_{co}\ if\ a_i = c > T$
$N_{tr}\ if\ (a_i = 1,2,\ ...,T)$
$\therefore\ T_{0,i} = a_i - 1$

$w_i = (w_{i1}, w_{i2},\ ..., w_{it})'$
$w_i : w_i(a_i), \begin{cases} w = 0\ \text{if}\ t < a_i \\ w = 1\ \text{if}\ t \geq a_i \end{cases}$
$W_{(N\times T)} = w_1, w_2,\ ..., w_N$

Assumption 1(Cross-Sectional Stable Unit Treatment Value Assumption: SUTVA)

- 횡단면적 파급효과를 배제하고, 잠재적 결과 경로의 수를 크게 줄임

$y_{it}(W_{(N\times T)}) = y_{it}(w_i) = y_{it}(w_i(a_i)), \forall\, i,t$

- 단위(i)의 잠재적 결과는 단위(i)의 처치상태에 대한 함수로 정의될 수 있음
- 예컨대, 통독의 효과 연구에서, 통일이 다른 국가의 경제성장에 영향을 미친다는 가정을 배제하는 것임(이것은 매우 강한 가정임)
- 다른 한편, 선거제도 변경 효과 연구에서, A 주(state)의 선거제도 변경 법률의 채택은 B주의 선거제도 변경 법률안의 채택여부와 상관없이 B주의 투표율에 영향을 미치지 않는다.

[Assumption 1 + Assumption 4 => Strict Exogeneity 엄격한 외생성]

$\{Y_{it}(0), Y_{it}(1)\} \perp\!\!\!\perp D_{is} | X_i^{1:T}, \alpha_i, f^{1:T}, \forall\, i,t,s$

D_{is}: 단위(i)가 시간(s)에서의 처치상태(통제집단 또는 처치집단 여부)

$Y_{it}(0)$: 통제집단 상태에서의 잠재적 결과 $Y_{it}(1)$: 처치집단 상태에서의 잠재적 결과

∴ 처치집단 선정은 기준선에서 이미 결정되며, 결과 실현과는 무관하다.

- 과거 결과가 현 결과에 영향 없음, 피드백 효과없음, 이월효과 없음 가정함

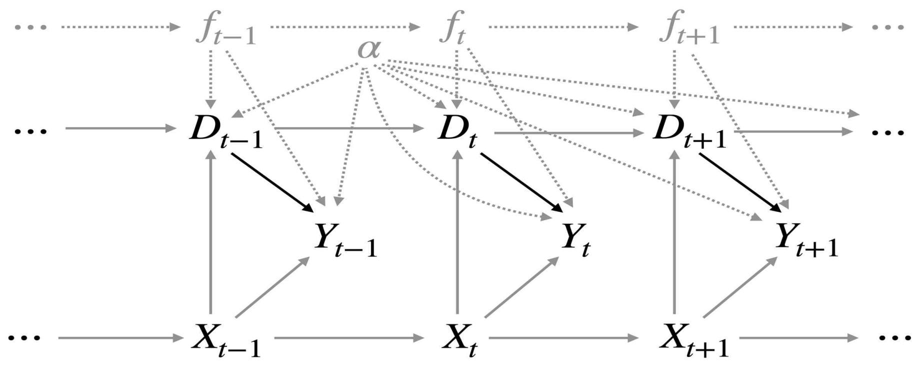


그림 3 DAG for DGPs Under Strict Exogeneity

Assumption 2(No Anticipation)

$y_{it}(a_i) = y_{it}(c), for\ t < a_i,\ \forall i$

counterfactual outcome: $y_{it}(c)$

Estimands

$\delta_{it} = y_{it}(a_i) - y_{it}(c), for\ a_i \le t \le T$

ATT(average treatment effect on the treated) for a duration of p periods:

$$\delta_p = \frac{1}{N_{tr,p}} \Sigma_{i:\, T-p+1 \le a_i \le T} \delta_{i,\, a_i+p-1}$$

$$Y(0)_{(N\times T)}\ under\ W=0\ (i.e.,\ a_i = c,\ \forall i)$$

$$Y(0)_{(N\times T)} \in \begin{cases} S_0 \equiv \{(it)|w_{it}=0\}\ y_{it}(c)\, observed \\ S_1 \equiv \{(it)|w_{it}=1\}\ y_{it}(c)\, missing \end{cases}$$

$$\therefore\ Y(0) = Y(0)^{obs} + Y(0)^{mis}$$

$$X_{(T\times p_1)} = \{X_1, X_2, \ldots, X_N\}, X_i = (X_{i1}, X_{i2}, \ldots, X_{iT})'$$

2. Assignment Mechanism:Rubin et al.(2010)

$$Pr(Y(W)^{mis}|X, Y(W)^{obs}, W)$$

$$\begin{aligned} Pr(Y(0)^{mis}|X, Y(0)^{obs}, A) &= \frac{Pr(X, Y(0)^{mis}, Y(0)^{obs})Pr(A|X, Y(0)^{mis}, Y(0)^{obs})}{Pr(X, Y(0)^{obs}, A)} \\ &\propto Pr(X, Y(0)^{mis}, Y(0)^{obs})Pr(A|X, Y(0)^{mis}, Y(0)^{obs}) \\ &\propto Pr(X, Y(0))\Pr(A|X, Y(0)) \end{aligned}$$

Assumption 3(individualistics assignment and positivity) no policy diffusion effects

$$Pr(A|X, Y(0)) = \prod_{i=1}^{n} Pr(a_i|X_i, Y_i(0)), 0 < Pr(a_i|X_i, Y_i(0)) < 1\ \forall i$$

Assumption 4(Latent ignorability) implies parallel trends assumptions

a vector of latent variables $U_i = (u_{i1}, u_{i2}, \ldots, u_{iT})$, $u_{it} = \gamma_i \bullet g(t)$

$Pr(a_i|X_i, Y_i(0), U_i) = Pr(a_i|X_i, Y_i(0)^{mis}, Y_i(0)^{obs}, U_i) = Pr(a_i|X_i, U_i)$

==> we will extract U_{it} from $Y_i(0)^{obs}$.

parallel trends assumptions: $u_{i1} = u_{i2} = u_{i3} = \ldots = u_{iT} = u_i$

Assumption 5 (Feasible data extraction)

$$U_{(N\times T)} = (U_1, U_2, \ldots, U_N)$$

$$U = \Gamma' F$$

$$F_{(r\times T)} = (f_1, f_2, \ldots, f_T)$$

$$\Gamma_{(r\times N)} = (\gamma_1, \gamma_2, \ldots, \gamma_N)$$

3. Posterior Predictive Inference

$X^{'}; X \,\&\, U$

$$Pr(Y(0)^{mis}|X^{'}, Y(0)^{obs}, A) = \frac{Pr(X^{'}, Y(0)^{mis}, Y(0)^{obs})Pr(A|X^{'}, Y(0)^{mis}, Y(0)^{obs})}{Pr(X^{'}, Y(0)^{obs}, A)}$$
$$\propto Pr(X^{'}, Y(0)^{mis}, Y(0)^{obs})Pr(A|X^{'})$$
$$\propto Pr(X^{'}, Y(0))$$

Assumption 6 (Exchangeability)

$\{(X_{it}^{'}, y_{it}(c))\}$is invariant to permutations in the index "it".

By de Finetti's theorem(de Finetti, 1963)

$$Pr(Y(0)^{mis}|X^{'}, Y(0)^{obs}, A) \propto Pr(\{(X_{it}^{'}, y_{it}(c))\})$$
$$\propto \int \left(\prod_{it \in S_1} f(y_{it}(c)^{mis}|X_{it}, \theta^{'}\right)\left(\prod_{it \in S_0} f(y_{it}(c)^{obs}|X_{it}, \theta^{'}\right)\pi(\theta)\, d\theta$$

<posterior predictive distribution> <likelihood>

θ: the parameters that govern the DGP of $y_{it}(c)\, given\, X_{it}^{'}$, $\theta^{'} = (\theta, U)$

4. Estimation Strategy(추정방법)

$O = \{ (i,t)|D_{it} = 0 \}, M = \{ (i,t)|i \in T, D_{it} = 1\}$ O=Observed, M=Missing

(1) "O"(통제집단)를 이용하여, $Y_{it}(0) = f(X_{it}) + h(U_{it}) + \epsilon_{it}$를 추정하는 $\hat{f}, \hat{h}$를 구함

(2) 처치집단 개체에 대해 반사실적 결과($Y_{it}(0)$)를 예측함

$\hat{Y}_{it}(0) = \hat{f}(X_{it}) + \hat{h}(U_{it})$ for all $(i,t) \in M$

(3) δ_{it}를 추정함 $\widehat{\delta_{it}} = Y_{it} - \hat{Y}_{it}(0)$ for all treated observation $(i,t) \in M$

(4) $\hat{\delta}_{it}$를 활용하여, ATT, ATT(s)를 추정함

$$\widehat{ATT} = \frac{1}{|M|}\Sigma_M \hat{\delta}_{it},$$

$$\widehat{ATT}_s = \frac{1}{|S|}\Sigma_{(i,t) \in S}\hat{\delta}_{it}, S = \{(i,t)|D_{i,t-s} = 0, D_{i,t-s+1} = D_{i,t-s+2} = ... = D_{it} = 1\},$$

Ⅲ. TSCS 접근방법 2 (순차적 무시성: Sequential Ignorability Regime)

1. Key Assumption

$\{ Y_{it}(0), Y_{it}(1) \} \perp\!\!\!\perp D_{it} | X_i^{1:t}, Y_i^{1:t-1}, \forall i,t$

$X_{it}^{1:t}$: 시간(t)까지의 공변인들의 역사

$Y_i^{1:(t-1)} = \{ Y_{i1}, Y_{i2}, \ldots, Y_{i,(t-1)} \}$: (t-1) 시간까지의 종속변수의 역사

- **순차적 무시성; 처치집단 선정은 모든 과거 정보(공변인과 종속변수 포함)를 포함하여 무시할 수 있음**
- **피드백 효과 인정(** $Y_{t-1} => D_t, X_{t-1} => D_t$ **)**
- **과거의 결과가 직접적으로 현재의 결과에 영향을 미침(** $Y_{t-1} => Y_t$ **)**
- **이월효과 인정(** $D_{t-1} => Y_t$ **)**

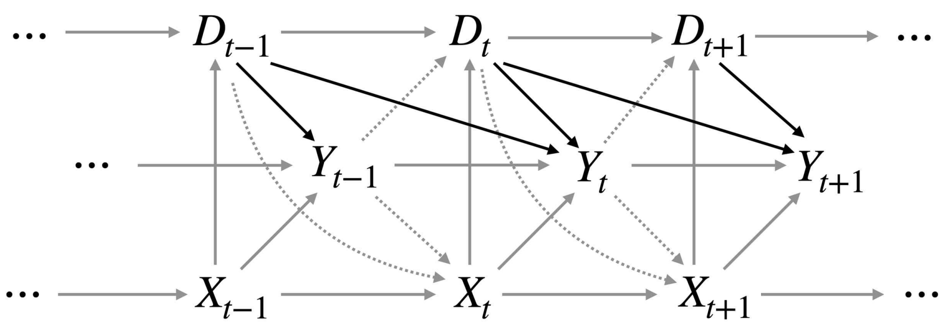


그림 4 DAG for DGPs under Sequential Ignorability

2. Ideal experiment: sequential randomization

3. examples: Lagged Dependent Variable Models(LDV), Autoregressive Distributed Lag Models(ADV), Marginal Structural Models(MSM)

4. PanelMatch (Imai, Kim & Wang, 2021)
 - 1 step: find matched units
 - 2 step: matching or reweighting
 - 3 step: block bootstrapping. compute ATT
 - 한계는, 데이터가 줄어듦

4. MSM(Marginal Structural Models) - epidemiology & biomedical sciences
 - Blackwell & Glynn(2018), Kurer (2020)